



## Cuprate Superconductors: Structure and Mechanism

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Received: 05 Apr 2024; Revised: 12 June 2024; Accepted: 22 June 2024; Available online: 10 July 2024

### Abstract

Superconductivity is most amazing phenomena in condensed matter physics, at first Superconductivity was limited to low temperature but latterly there is filed developed to find High Temperature Superconductors (HTSC). Among those, Cuprates have remarkable impact. These cuprates first identified as semiconductor material and later it turned to superconducting material. This paper we discussed chemical structure of cuprate superconductors, their behavior in brief and theories which suggest to explain it including BCS theory, t-j model, Hubbard model and novel theory which use to explain superconductivity using Heitler-London centers.

**Index Terms**— BCS theory, Cuprate Superconductors, Models of Superconductivity, Superconductivity

### Introduction

The traditional human fantasy of perpetual mobility is realized by superconductivity, which allows electric current to flow without resistance. The resistance ratio that exists between normal-conducting and superconducting states has been measured to be greater than 10<sup>14</sup>, which is at least as great as the difference between a typical insulator and silver, the best normal-conducting material. When cooled below a specific critical temperature ( $T_c$ ), certain materials exhibit the phenomena of superconductivity, which results in exactly zero electrical resistance and the expulsion of magnetic fields[1]. Although the phenomenon of superconductivity expels magnetic flux, the family of copper oxides has a very high superconducting (SC) temperature (without external pressure applied). Additionally, the superconducting phase is close to an Antiferromagnetic (AF) phase with a transition temperature that is three times higher than the superconducting one[2].

Early in 1986, the highest temperature at which superconductivity could be achieved was below 30 K [3], and the first cuprate superconductor which at first showed semi-conductivity[4] discovered in 1986 Bednorz and Muller's discovery of superconductivity in LaBaCuO cuprate ceramics at 30 K changed the path of a superconducting phenomenon that had previously only been observed at very low temperatures [up until 1986, the greatest value of  $T_c$  was only observed at 23 K in Nb<sub>3</sub>Ge. As typical materials these undoped copper oxide are also insulators but when doped, the latter can transform from poor metals in their natural condition to high-temperature superconductors (HTSCs) when the temperature is lowered[5]. The maximal critical temperature range for the cuprates systems have been shown to be between 126K and

200K so far[6].

The copper oxides would have appeared to be the least likely materials to look for superconductivity based on the theory of 'conventional' superconductors because they are such poor conductors at room temperature that they can hardly be classified as metals. Even if their chemical composition is slightly changed, they transform into highly insulating anti-ferromagnets. Magnetism and conventional superconductivity appear to be incompatible types of order because magnetism results from strong repulsive interactions between electrons and conventional superconductivity from induced attractive interactions[4]. The superconducting cuprates differ significantly from normal superconductors in such a way that they are doped oxides that behave like terrible metals rather than traditional metals. Superconducting pairing frequently involves doped holes rather than electrons because they act as quasiparticles that pair up and behave like Cooper pairs but have the opposite charge[7].

If the Bardeen-Cooper-Schrieffer (BCS) theory can be used to explain a superconductor, then it is typically regarded as a conventional superconductor. Type-I or type-II conventional superconductors are both possible. Conventional superconductors make up the majority of the atomic ones. If a superconductor cannot be explained through BCS theory, then it is referred to as an unconventional superconductor. Since the BCS theory cannot explain high temperature superconductors, they are typically referred to as unconventional superconductors [2]. For much more basic causes, high-Tc cuprates are unique in that they show unanticipated physical events occurring in a situation where neither the conventional band model nor the BCS theory can explain it[8]. For a long time, it was believed that all superconductors were constrained by BCS theory, which held that superconductivity could not exist in materials beyond 30 K. It wasn't until 1986 that Johannes G. Bednorz and Karl A. Muller made the discovery of a brand-new class of cuprate superconductors that went above this BCS theory cut off[7]. These cuprate compounds exhibit type II superconducting (SC) states, which according to the BCS/London framework, means that the coherence length is less than the penetration depth. In contrast to elemental, phonon-mediated superconductors, where the SC gap is always s-wave, cuprates have a second critical gap that is d-wave with very high second critical fields of the order of 100 T[9].

Additionally, a cuprate was found to have a Tc that was higher than the 77 K boiling point of liquid nitrogen. This made it clear that superconductivity applications were more likely to be practical and practicable in the foreseeable future. High-temperature superconductors are defined as superconductors having a critical temperature higher than the boiling point of liquid nitrogen. It's crucial to remember that type-II superconductors are the only high-temperature superconductors. The record for the greatest critical temperature at atmospheric pressure is currently held by cuprate superconductors. Other materials have recently shown even higher critical temperatures, but only under very high pressure [7].

Strongly coupled electron systems have made great progress thanks to the finding of superconductivity in cuprates. Cuprates exhibit anomalous phases and phenomena, such as the Mott metal-insulator transition, pseudo gap phenomena, antiferromagnetic (AF), and charge-density-wave (or stripe) phases, in addition to the high superconducting critical temperature Tc. Recently, experimental observations of ferromagnetic fluctuation and nematic order have also been made[10]. For more than 30 years, cuprate high-temperature superconductors (HTSC) have become the focus of intensive research. Yet there is still no agreement on the superconductivity's underlying process. According to conventional opinion, doping a strongly correlated antiferromagnetic (AFM) insulator is the source of the strange and exceptional features of the cuprates. Beyond the dome of superconductivity (SC), highly over doped cuprates are regarded as typical Fermi liquid metals [11]. Before considering major theories, a brief idea about the chemical structure of cuprates

is given below.

## Chemical Structure of Cuprates

The division of cuprates are into two components—the blocking layers and the  $\text{CuO}_2$  planes—is a relatively typical property. Only the  $\text{CuO}_2$  planes exhibit superconductivity, and the blocking layers provide charge carriers to the  $\text{CuO}_2$  planes. All has the same quality. Two-dimensional superconducting  $\text{CuO}_2$  sheets inside the structures serve as the superconducting plane charge carriers. In the  $\text{CuO}_2$  planes, contributions from the Cu  $3d_{x^2-y^2}$  and O  $2p$  orbitals—where the orbitals are those with their lobes pointing at the surrounding Cu sites—dominate the states close to the chemical potential. Since there is only one hole per Cu site, if one merely takes into account the nearest-neighbor hopping energy, one would anticipate seeing a partially filled hybridization band [12]. The  $\text{Cu}^+$  ions are in a nominal  $(3d)^9$  configuration and have a magnetic moment in the undoped  $x = 0$  state. The undoped system is an insulating antiferromagnet with a charge transfer gap below a Neel temperature. More holes are added to the  $\text{CuO}_2$  layer when some of the La is replaced by Sr. The antiferromagnetic order is inhibited when the system is doped with holes, and as a result, the system eventually turns metallic and superconducting. There is an orthorhombic to tetragonal lattice transition and a spin-glass-like area in addition to the insulating-metal crossover [13]. The  $\text{CuO}_2$  planes are a common structural component of all cuprate superconductors, whether they are tetragonal or orthorhombic[14]. There is a minor orthorhombic distortion in the cuprate superconductors [13]. Because of structural distortion in the  $\text{CuO}_2$  layers, which causes a preferred axis along one of the Cu-O directions to spin by 90 degrees from one layer to the next, the stripes are able to be pinned to the lattice [15]. The substance is made up of insulating spacer layers separating  $\text{CuO}_2$  planes, where each Cu ion is fourfold coordinated with an O ion. The exception to this rule is YBCO, where one of the spacer layers is a metallic CuO chain layer [16]. However, one of these oxygens is missing in YBCO, and the apical oxygens are entirely gone in other formations, such as those made of electron-doped cuprates[16]. Electric conduction in cuprate superconductors results through the coupling of charge carriers into copper pairs[2].

The oxygen doping is the most common way to change the  $T_c$  in high  $T_c$  cuprate superconductors at ambient pressure. Contrary to elemental low temperature superconductors, dopants that change the electronic structure close to the Fermi surface determine the amount of  $T_c$  in high temperature superconductors [17]. The temperature and doping dependency of charge order has been thoroughly investigated in a number of researches on various cuprate families. As a result, charge density modulations are evident at all examined temperatures and in a wide doping range, with a shorter correlation length compared to low temperature charge density waves[18]. The experimental discovery of charge order (CO) in the underdoped regime of cuprates has rekindled interest in the topic. It all began with Scanning Tunneling Microscopy (STM)s discovery that when a magnetic field is applied, vortices exhibit modulations. The NMR (Nuclear Magnetic Resonance) experiment was able to quickly detect these modulations and show that the signal is non-magnetic, pointing to CO as the primary source[3]. We have reached crossroads where the theoretical and empirical paths diverge: either hole-doped cuprates have topological order, or they are fundamentally similar to electron-doped cuprates but they have considerably shorter Anti Ferromagnetic correlations. Regardless, this is a ground condition that is amazing and has never been seen before [19].

Despite the fact that the superconducting transition temperature varies greatly between structures and is

typically higher the more CuO<sub>2</sub> planes per unit cell there are, most researchers have concluded after years of research that the primary c-axis effect is simply adjusting the electronic structure of the CuO<sub>2</sub> plane [16]. Every cuprate superconductor has a reasonably high c/a lattice (Lattice parameters) constant ratio, which distinguishes it from other superconductors [14]. These c/a ratios correspond to a flattened Brillouin zone with the fundamental symmetry characteristics of a square or rectangular lattice in the k-space presentation [14]. The removal of oxygen p-orbitals' electrons by hole doping has been demonstrated by direct experimental evidence [20].

When there is one electron on each Cu site of a cuprate's CuO<sub>2</sub> plane, a Mott insulator that is immobile occurs due to the intense repulsive contact between the electrons in cuprates. Electron mobility is restored by withdrawing electrons or introducing p holes (per Cu site), and at high enough p cuprates, well-behaved metals can be observed. The peculiar events take place in the intermediate regime, which is between the Fermi liquid at  $p > 0.3$  and the Mott insulator at  $p = 0$  [19]. The existence of "intertwined" order parameters corresponding to numerous distinct broken symmetry states that occur with similar energy and temperature scales, and which in some ways compete and in some ways cooperate strongly with one another, is another striking feature of the underdoped cuprate phase diagram in addition to the energy gap [15].

The anti-ferromagnetic Mott insulating phase, the superconducting phase, the strange metal phase, and the Fermi liquid phase are the four electronic phases found in cuprate superconductors that have been researched to date [17]. The classic theories of transportation valid within the Fermi liquid paradigm are defied by the extremely unique electric and thermal transport features demonstrated by tests. In fact, the "strange metal phase" exhibits linear T resistivity up to a thousand Kelvins, satisfying the Wiedemann-Franz equation [3]. When coherent single particle hopping between right next CuO<sub>2</sub> planes is inhibited in the normal state, interlayer tunneling mechanism is most effective. Recent theoretical work has stressed that the interlayer hopping should appear as a gap at the center of the two-dimensional (2D) Brillouin zone (BZ) because it is also encoded in the cuprates' plasmon spectrum. More particular, in layered systems like cuprates, the plasmon dispersion (for small q) neither follows the  $q^2$  behavior of isotropic 3D metals nor the conventional  $\alpha\sqrt{q}$  dependency of 2D metals. Instead, a plasmon spectrum with a number of acoustic branches that disperse linearly for small q and one optical branch results from poorly screened inter layer Coulomb interaction between the CuO<sub>2</sub> planes [21].

## Properties

One of the unique characteristics of cuprate high temperature superconductors is their linear-in-temperature electrical resistivity. These materials can be tuned to a carrier concentration where  $(T) = 0 + AT$  in a wide temperature range by chemical doping [22]. For temperatures  $T > T'$ , where  $T'$  is a doping-dependent cross-over temperature that drops as a function of doping and disappears at a threshold doping p, the linear-in-T resistivity is universal for the underdoped cuprates. Within the range of  $0.19 < p < 0.40$ , the precise value of p varies significantly between cuprate families [22]. While for  $T' = 0$  the stripes were filled and superconductivity was absent, the general appearance of stripe orders on larger systems with non-integral numbers of pairs suggests that pairs fluctuate between stripes, promoting long-distance phase coherence and thus superconductivity [23].

It proves the presence of itinerant ferromagnetic order at temperatures below 4 K in the over doped, non-SC, cuprate LCCO ( $\text{La}_{2-x}\text{Ce}_x\text{CuO}_{4-d}$ ). This implies a conflict between ferromagnetism and d-wave

superconductivity as a result of the existence of a ferromagnetic quantum critical point at the end of the superconducting dome. Other unexplained characteristics of the over doped cuprates, such as the reduction in TC beyond optimal doping and the anomalous loss of superfluid density, may be influenced by this competition [11]. Later Anderson and co-workers proposed that T<sub>c</sub> is not a single-plane feature and interlayer Josephson tunneling of Cooper pairs greatly amplifies the T<sub>c</sub> of cuprates regardless of the precise type of pairing glue [21]. By freezing the global phase at T, a particular Higgs mechanism entangles the two preformed pairs, and a gap appears in the fermionic excitation spectrum. The entanglement causes p-p and p-h pairings to compete fiercely with one another. The two amplitudes and the relative phase of the fields corresponding to the two pairs change throughout time. The relation between the amplitude fluctuations is given by the constraint  $|z_1|^2 + |z_2|^2 = (E^*)^2$ . A special series of occurrences then take place as the temperature drops after that. Lower temperatures, T<sub>co</sub> and T'<sub>c</sub>, respectively, cause the amplitudes of the p-h and p-p pairs to condense. At T = T<sub>c</sub>, a second Higgs mechanism takes place, and the superconducting and bond-excitonic orders both acquire phase coherence, resulting in a phase that resembles a "super-solid." Thus, the rich phase diagram of cuprates contains various temperature lines [24] and also phase diagram is of Cuprates has interesting facts.

There are three main ideas about phase diagram of cuprate superconductors, first one is observations that a Mott transition is imminent when examine the phase diagram of cuprates. This metal-insulator transition suggests a strong Coulomb repulsion of the order of 1 eV at each site. The electrons are completely localized on the various lattice locations at half filling, creating an insulator. Anti-ferromagnetic order coexists with this insulator production and lasts until a temperature of roughly 700 K. Strong electron correlations are caused by the existence of strong Coulomb energy, the biggest energy scale in the system, to the extent that this region of the solid state is known as a strongly correlated electron system. And the Fermi liquid's phenomenology, which depicts typical metals at low temperatures, can be found starting with a big enough oxygen doping in the phase diagram, which is the second remarkable feature. This has led to the theory that a zero-temperature phase transition, also known as a Quantum Critical Point (QCP), is concealed beneath the superconducting dome and that the PG phase is connected to a "broken" symmetry of the system corresponding to this QCP. In quantum materials physics, the occurrence of a QCP under a superconducting dome is fairly frequent. The final third strategy is to think of the intermediate oxygen doping phase as a phase with a lot of swings. There are a number of causes for this. The system is extremely anisotropic and quasi-bidimensional, which causes quantum fluctuations, on the one hand. The phases of the various particles shift during a localization transition, however, because the PG phase is close to a metal-insulating conversion (the Mott transition) [3]. The original hypothesis of Bednorz and Muller was that some oxides could support Jahn-Teller type composites, which are composed of an electron and a local lattice distortion and can pass through the lattice as a whole. This could result in a very strong electron-phonon coupling. While it is undeniable that lattice distortion and strong electron-phonon coupling play a significant role in the high T<sub>c</sub> of the cuprates, other non-trivial (such as magnetic) phenomena have since been observed that also seem to have a significant impact on the T<sub>c</sub>[25]. And the spin and charge responses of the underdoped cuprate superconductors also have a close relationship[26].

## **Suggested Models to describe Cuprate Superconductivity**

Superconductors made of cuprates are thermodynamically unstable. With the development of new cuprate superconductors, its mechanism remains unstable. Pairing, or non-retarded or retarded interactions,

constituted the foundation for the mechanism of high  $T_c$  superconductivity. There is still debate about this concept. The energy scales, sometimes referred to as the superconducting gap and the pseudo gap, are another well-known theory on cuprates. The quasi-particle spectrum, which had been proposed to investigate the excitations of the superconducting cuprates, is initiated at these energy scales. Because there is disagreement over the validity of pseudo gap, its success was short-lived [6]. So considering various models that suggested in explaining superconductivity, the initial theorem was BCS theory. In the late 1950s, the BCS hypothesis led to conceptual advances and provided an incredibly successful framework for understanding conventional superconductors. The fundamental realization is that the electrons collectively form 'Cooper' pairs and concurrently condense in a manner reminiscent of how bosons condense into a superfluid state. The fact that, despite the strong direct Coulomb repulsions, the relatively weak interactions between electrons caused by the coupling to the lattice's vibrations (phonons) can pair the electrons at energies below the average phonon energy is fundamental to the BCS mechanism. This led to the widespread assumption that the superconducting transition temperature ( $T_c = \text{critical temp}$ ) of conventional superconductors could never rise above 30 K. However, this limit has since been revised upwards following the discovery of superconductivity with a  $T_c$  of 39 K in the simple metal  $MgB_2$  when conditions align to favor the electron-phonon mechanism. However, this is still much lower than the copper oxides' maximum  $T_c$ . As the properties of copper oxides were studied with ever-increasing sensitivity and precision, it became apparent that much of the well-known quantum theory of the electronic properties of solids, which has been extraordinarily successful in accounting for the properties of conventional metals and superconductors, completely misses the mark on many features of copper oxides and, more generally, of a wide range of "highly correlated electron systems" of which the copper oxides are a subset [4]. The BCS theory's electron-phonon interaction is still considered crucial, and this fundamental process driving superconductivity is believed to remain constant. Therefore, the idea that strong electron-phonon interaction induces high- $T_c$  superconductivity has been the foundation of the majority of research. Numerous investigations have concentrated on clarifying the configurations of electrons and their robust correlations in high- $T_c$  superconductors during this phase [27]. Because of the low carrier density in cuprates, phonons significantly aid in Coulomb screening. The Cu-O bond stretching mode is the phonon branch with the greatest influence.  $O_2$  forms a symmetric breathing mode around  $Cu^{2+}$  at  $q = (\pi, \pi)$ , and half-breathing modes are present at  $(\pi, 0)$  and  $(0, \pi)$ . The strongest phonon softening in cuprates like LSCO and YBCO are in the Cu-O bond direction, and it gets stronger as one proceeds from the zone center to the zone boundary, according to neutron scattering tests. Close to the charge-ordering wave vector, there is a significant dip [12]. Although BCS theory is consistent with the lack of a significant chemical potential change with temperature, this does not necessarily mean that BCS theory is sufficient to explain the emerging low energy features of cuprates [28].

Contrary to BCS theory, which states that the zero-temperature should not depend on  $T_c$  at all in a clean system but instead be simply proportional to the carrier density (which rises as the system is doped), it has been discovered that the zero-temperature superfluid density ( $\rho_s$ ) at various doping is proportional to the  $T_c$ . Although  $s(0)$  and  $T_c$  might have a quasi-proportionality in the dirty limit [29]. The Landau-Fermi liquid picture, which served as the foundation for the BCS paradigm, also significantly differs from the cuprate superconductors normal state. The formation of the so-called pseudo gap well above  $T$  is one of the unusual features in this situation [30]. Understanding cuprate superconductors needs thorough consideration of the p- and d-orbital degrees of freedom [31].

However, the cuprate situation is distinct. The O ion is in a p 6 configuration (O) and the Cu ion is in a d 9

configuration ( $\text{Cu}^{++}$ ) in the solid. The Cu 3d energy level is above the O 2p energy level but reasonably near to it. The solitary 3D hole in the layered perovskites has  $dx^2y^2$  symmetry due to the Cu ion's tetragonal surroundings. In this instance, the bonding-antibonding breaking comprising the quantum mechanical combination of the planar O  $2p_x$  and  $2p_y$  orbitals and the Cu 3d  $x^2 - y^2$  orbital is the dominant energy. As a result, one is left with a half-filled band in the parent structure that is the antibonding combination of these three orbitals, with the bonding and non-bonding states. In the parent compound, this copper-oxygen antibonding band "Mott-Hubbardizes," creating an insulating gap of order 2 eV (the effective U is decreased due to the Cu-O orbital mixing). The complex electronic structure ultimately results in a single 2D energy band close to the Fermi energy, which explains why cuprates are so intriguing from a theoretical standpoint.

Moreover, if the limit of the huge U is taken, this "one band Hubbard model" can be further condensed. The impact of U becomes virtual in this situation and the upper Hubbard band is projected out, resulting in a super exchange contact between the Cu spins, J ( $t^2/U$ , where t is the effective Cu-Cu hopping caused by intervening O sites). The Pauli exclusion principle prevents two parallel spins from occupying the same Cu site, while antiparallel spins can, resulting in an energy saving of  $t^2/U$  from the second-order perturbation theory. This is easily understood. The "t-J" model, as it is known, is the simplest model for cuprates [16]. What is remarkable is the nearly unanimous agreement that the two-dimensional Hubbard model (or various t-J as well as related phenomenological models based upon the interaction of quasi-particles with antiferromagnetic spin-fluctuations) will enter a  $dx^2 - y^2$  state if it becomes superconducting when doped close to half-filling [13].

In the Hubbard model conceptualization several computations indicate that the on-site Coulomb interaction can lead to superconductivity when the system is slightly doped away from half-filling. Strong-coupling variational calculations and weak-coupling perturbation theory have both been used in these researches. Both Monte Carlo simulations of bigger clusters and accurate diagonalization analyses of small clusters are included. They range from phenomenological methods to preserving approximations [13]. The Hubbard model, which distinguishes between electron- and hole-doping by hopping to the next neighbor, does indeed reflect the key characteristics of the charge, magnetic, and pairing ordering [23]. A qualitative shift occurs when the Hubbard model is applied to hole-doped cuprates by raising  $U/t$ . Indeed, a pseudo gap phase arises at  $T = 0$  when  $U/t$  exceeds a critical value of around 6. At  $T = 0$ , it begins as a crossover with decreasing T rather than a transition with decreasing p, possibly of the first order. According to recent calculations, and in line with experiment, the Hubbard model maintains the inequality  $p^* \leq p_{FS}$  [19]. Despite considerable progress, there is a growing consensus in the field that the single band Hubbard model cannot adequately explain the physics of the hole doped cuprates [20]. The local Coulomb repulsion has a pervasive impact on normal state parameters as well as magnetic and superconducting phases in the specific case of the high- $T_c$  cuprate superconductors. The Fermi surface is made wider by the Hubbard U, which increases the quasiparticle effective mass. Additionally, it establishes the magnetic super exchange scale and, in a spin-fluctuation-mediated model, directly determines the critical temperature and superconducting pairing. The Hubbard U in cuprate superconductors can be altered by ultrafast lasers, although this is still an unsolved experimental and theoretical issue [32]. It is possible to combine a Cooper pair and explain the formation of the pseudo-gap using the RVB (Resonance Valance Band) model or Hubbard-several theoretical articles and a model.

In addition to the anomaly metal phase and the transition temperature  $T_0$  at which the anomaly metal phase arises, analytically discuss and interpret experiment results [8].

The fact that Cooper pair sizes range from around 10 to 30 Å indicates that pairing is local in Cuprates. This information comes from measurements of indirect penetration depth. Even smaller pair sizes may actually exist. Compared to the BCS superconductors, where the value is on the order of 1000 Å, this is many orders of magnitude less. The fact that the pairs are small does not preclude the employment of mean-field methods in the case of cuprates. The solution by V. J. Emery using mean-field BCS yields a reasonable value for  $T_c$  [20]. More constrained,  $dz^2$  orbital-based band independent of superconducting. The latter promotes the migration of mobile carriers and hence increases superconductivity because it has a higher  $z$ , or more precisely, a weaker electron correlation effect. These two aspects are considered in the current four-band d-p model, which goes beyond the usual one-band and even three-band d-p models. Because of this, this model is thought to be the minimal one that can adequately represent the material dependency of cuprate superconductors, acting as a crucial guide for the development of novel materials with greater  $T_c$  [31]. When the many-body calculation approach applied to a multi-orbital model results in different site energies for each orbital due to the interaction effect. However, the energy band of the tight-binding model, which was constructed from the first-principles calculation, already accounts for these fluctuations in energy. It is important to approach this "double counting problem" cautiously, especially in the d-p model. [31]. Initially, the d-p model also referred to as the Emery model which studied as a multiorbital effect model. Cu  $dx^2y^2$  orbitals and O  $P_x/P_y$  orbitals make up its composition. These models have been well studied in terms of stripe features, pseudo gap phenomena, material dependency on  $T_c$ , and other aspects. These models are not able to sufficiently describe the special characteristics of cuprates, even though they have managed to capture some of the unusual characteristics proposed a four-band d-p model, comprising the O  $P_x$  and  $P_y$  orbitals and the Cu  $dx^2y^2$  and  $dz^2$  orbitals, to provide a coherent explanation of cuprate superconductors. It has shown that model correctly describes two important features of the Fermi surface contribution provided by the Cu  $dz^2$  orbital as well as the slight energy difference between the Cu  $dx^2y^2$  and O  $P$ - orbitals, based on the Variational Monte Carlo (VMC) approach [31]. The on-demand modification of the Coulomb repulsion in correlated materials may pave the way for the identification of novel non-equilibrium states of matter, such as  $\nu$ -paired superconductivity in driven Mott insulators and fragile quantum spin liquids in frustrated magnets. Exchange contacts may be impacted by a dynamical Hubbard  $U$  in the former, which could lead to the antiferromagnet changing into an entangled spin liquid state. By adjusting the Hubbard  $U$  in a driven-dissipative Mott insulator, the latter will dynamically vary the energy spectrum and doublon number of the driven Hamiltonian, allowing long-range staggered superconducting correlations to form. [32]. Also doping a spin-liquid state was found to naturally produce a d-wave superconductor [33]. But it is currently unclear if the spin channel by itself is the source of the superconductivity in cuprates, despite the general consensus that antiferromagnetic spin fluctuations are crucial for the superconducting pairing. For example, the significance of electron-phonon interaction remains controversial [21].

The chemistry of copper oxides increases the electron Coulomb repulsions. The two-dimensional copper oxide layers are separated by ionic, electrically inert buffer layers. For every  $\text{CuO}_2$  unit cell, the stoichiometric "parent" molecule has an odd-integer number of electrons. In the states formed in  $\text{CuO}_2$  unit cells, an electron must be taken out of one site and added to another with a large amount of energy (the Hubbard  $U$ ), exactly as it would in a group of atoms that are well separated. As a result, there is an electron "traffic jam." The term "Mott insulator" refers to an insulator produced by this conventional jamming phenomenon. Nevertheless, an electron's spin orientation remains a dynamical degree of freedom. Virtual hopping of these electrons causes an antiferromagnetic interaction between neighboring spins via the Pauli



exclusion principle. The resulting simple (Ne'el) ordered phase occurs below room temperature, with the direction of the static magnetic moments on the Cu sites reversing from one Cu to the next[4]. Quantitative analysis was done on collective spin- and charge excitation in a microscopic model of high-Tc copper-oxides. These modes are widespread in the absence of long-range spin density waves and CDW orders, and they are present throughout a broad range of temperatures and doping levels. The main challenge in defining the underlying electrical states is their high correlation[34].

The anomalous features in cuprate superconductors were investigated as the physics of an effective single band crossing a Fermi surface long before multiorbital effects were considered. Effective one band theories, the Hubbard and t-J models, performed well in forecasting d-wave superconductivity but poorly in explaining the material dependency of Tc and the peculiar competing orders[31]. Also phonon anomaly is a type of characteristic phonon phenomena that appears uniquely in high-Tc cuprate superconductors. Many studies have looked into the connection between superconductivity and phonon abnormalities. Specifically, the YBCO B1g phonon anomaly appears slightly above Tc as the temperature decreases, and the B1g phonon frequency rapidly decreases below Tc due to softening. The B1g phonon anomaly is believed to be caused by the gap opening-induced depletion of the electron density of states, however the relationship between the two has not been established. It is necessary to have a greater understanding of superconductors and to explain how the B1g phonon anomaly affects superconductivity[27]. In many-body scenarios, the carriers in various materials interact with each other. Since the general band theory cannot be applied to high-Tc cuprates, this argument is particularly important. The many-body interactions of carriers result in several local temperatures  $T_i$  in the materials, where  $i$  is the index for a site. Stated otherwise, thermal equilibrium is only a reasonable assumption at a temperature of  $T_i$ . This makes it a paradigm for controlling interactions between several substances. A spherical shell with radius  $a_i$  has local temperature  $T_i$  and differential number  $dN$ . At the centre is also a macroscopic Boson. The pressure produced by the outermost particles in this sphere is equal to their kinetic energy in  $dN$ . On the other hand, force of expansion—which represents electrostatic energy or Coulomb interactions—is provided by the center macroscopic boson. Here, we include the additional expansion force of magnetic interactions between macroscopic bosons[8].

When a  $\text{CuO}_2$  surface is treated as the focus point and cooling is adequate to the extent that the wavelength of a hole is greater than the width of the surface, it is said that 2D is fully formed. This indicates that every hole self-rotates to form a circle, indicating that angular momentum conservation must exist on the surface. Because this rotating circle carries magnetic field energy, consider that a new particle has now been created. Remember that the particle creation implies a phase transition. This is relevant to the electron nematic phase. We shall hereafter refer to this new particle as a "macroscopic Boson"[8].

In a simple description of the commensurate resonance between the quasiparticle orbit of the highest Landau level caused by the magnetic field and the Density of States modulations, some physicists propose an alternative model of the Quantum Oscillation results on hole underdoped cuprates that does not require a Fermi surface reconstruction or a cloning procedure. In some observations and the lack of clear spectroscopic evidence of tiny electron-like pockets on hole under dope serve as the foundation for this model[35].

It is not implied that this BCS-BEC crosser is irrelevant in other materials just because it is insignificant in cuprates [36]The Fermi surface and general features of the electron dispersion observed in ARPES

experiments across the superconducting dome of cuprates, however, are largely consistent with predictions from band-structure calculations, suggesting that there haven't been any notable excursions toward the BEC limit[28]. Recently introduced theory suggest that the electronic structural deformation of the crystal by the dopant ion is assumed to be local in nature in the proposed model of hetero valently doped cuprates. It first treats undoped cuprates as gap-filled charge-transfer (CT) insulators. The localization of the doped carrier in the close proximity to the dopant as a result of the formation around the doped charge of a trion complex—the bound state of the doped carrier and CT excitons that arise under the influence of this charge in the surrounding  $\text{CuO}_4$  plaquettes, which we refer to as CT plaquettes—determines the local nature of the deformation of their electronic structure. This can happen if the doped charge suppresses the initial charge-transfer gap  $ct' < ct$  in nearby  $\text{CuO}_4$  plaquettes to a level that allows for the formation of CT excitons in these plaquettes. The location of dopants within the crystal lattice dictates the geometry of the arrangement of trion complexes and related CT plaquettes, which varies typically for every chemical. The majority of the anomalies in the superconducting properties of cuprates that were observed at dopant concentrations inside the superconducting dome, as well as the precise location of the domes on phase diagrams of compounds, were demonstrated to be explainable and highly accurate within the parameters of a straightforward geometric model that described the cluster structure of the superconducting phase[37].

## Discussion

Cuprates show most amazing material properties due to their crystal structure. With superconducting layer and blocking layers which carries charges gave this superconducting property to cuprates and also, there are lot of cuprite perovskites materials which can be use as superconductor. There are research projects ongoing to conduct material analysis and predict possibility of new types of superconducting material. However cuprates identified as first semiconductor material [1] but it took some time to find out superconductivity in it. Considering all the theories mentioned above finally it can be said that there is no perfect theory for describe cuprates. Nowadays scientists are developing relativistic BCS theory and it will give some close explanation for cuprate superconductors and also crossover of BEC-BCS. But there is a gap for universal type of theory which can explain all type of superconducting materials including Cuprates. These cuprates give hope for next generation room temperature superconductivity because their unique chemical structure which gives them good doping ability and adaptability. Moreover, cuprates can easily manufactured than other types of HTSC materials also lot of cuprate materials are not toxic as Iron arsenide. The findings of research in 2008 support theoretical explanations of the high- $T_c$  cuprate phase diagram that postulate the presence of an extra hidden quantum critical point close to the optimal doping [38].

For the first time Examination of the magnetic characteristics of the  $n=3-5$ , 20 multiphase cuprate superconductors  $\text{Bi}_{1.7}\text{Pb}_{0.3}\text{Sr}_2\text{Ca}_{n-1}\text{Cu}_n\text{O}_y$  that were created with SFAQ-T technology. Precursor phases were found close to  $T_c=107-160$  K. By examining the characteristics of the acquired dependencies and contrasting them with additional findings related to the processes around the critical temperature  $T_c$ , it is possible to deduce the presence of high-temperature superconducting precursor phases. For the first time, the comparison of torsional and vibrating reed magnetometries was examined in order to assess the  $T_c$  of the superconducting precursors in the multiphase HTSC Bi-Pb-Sr-Cu-O system made using the SFAQ-T technology [39]. It demonstrated that the outcomes from both techniques were sensitive to superconducting diamagnetism, which made it possible to identify new superconducting precursor phases in these samples

above bulk  $T_c$ . Furthermore, the vibrating reed spectroscopy method may be more advantageous than the low-frequency torsional spectroscopy method when it comes to studying the superconducting precursors in multiphase HTSC ceramics [39]. So, there is widely open door for study HTSC Cuprate magnetic behavior in quantum scales. Moreover, using RPS (Reduced Elastic Scattering Phase) we can easily understand certain features of unconventional superconductors, such as the various phases' dependence on the strength parameter  $c$  and the concentration of non-magnetic disorder[40]. Also, scientists have demonstrated that a number of peculiar traits that set underdoped cuprates apart from conventional superconductors seem to be common to both; these properties stem from the underdoped phase's cluster structure and the unique superconducting pairing mechanism in cuprates. This pairing mechanism is similar to the Little-Ginsburg mechanism and is thought to be mediated by the virtual dispersion of electrons into empty localized pair states created on pairs of nearby Cu cations during doping[37].

Most considerable fact is within a specific temperature range  $T_c < T < T^*$  finite isolated clusters can exist in both superconducting and normal states due to the cluster structure of the underdoped phase and the unique mechanism of cuprate superconductivity. These clusters randomly switch between these states as a result of fluctuations in pair state occupation with electrons [41].

## Conclusion

There are lots of hidden mysteries in Cuprates superconductors and it opens various kind of doors to modern experimental physicists and theoretical ones. And also applications in industrial uses are more advantages comparing to Iron Based Superconductors because lot of Iron based contains toxic materials[42] and also cuprates are easier to manufacture.so novel scientific research areas required to explore these magnificent materials including perfect theory for explain behavior of these type.

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